

List of Ph.D. Course work subjects that can be offered under  
Mathematics Group from 2024

Group 1		Group 2		Group 3		Group 4	
Subject Code	Name of the Subject	Subject Code	Name of the Subject	Subject Code	Name of the Subject	Subject Code	Name of the Subject
PHMA101	Algebraic graph theory	PHMA201	Fluid Dynamics	PHMA301	Computer fundamentals and programming in C	PHMA401	Ferro hydrodynamics
PHMA102	Partial differential equations	PHMA202	Numerical concepts in applied mathematics	PHMA302	Complex analysis	PHMA402	Integral transforms & calculus of variation
PHMA103	Solution of differential & partial differential equations	PHMA203	Applied mathematical methods	PHMA303	Mathematical models for bio fluids	PHMA403	Optimization models and applied Partial Differential Equations
PHMA104	Mathematical modeling for bio physical systems	PHMA204	Boundary layer theory	PHMA304	Probability models	PHMA404	Principles of lubrication
PHMA105	Advanced differential equations	PHMA205	Integral equations and boundary value problems	PHMA305	Theory of hydrodynamic lubrication	PHMA405	Calculus of vectors and fluid mechanics

PHMA106	Applied Numerical Analysis	PHMA206	Topology	PHMA306	Application Of The Graph Theory To Group Structures	PHMA406	Bio Fluid Dynamics
PHMA107	Number Theory	PHMA207	Linear Algebra	PHMA307	Advanced Fluid Mechanics	PHMA407	Applied Partial Differential Equations
PHMA108	Metric Spaces	PHMA208	Tensors And Differential Geometry	PHMA308	Discrete Mathematics	PHMA408	Probability And Mathematical Statistics
PHMA109	Special Functions And Integral Transform	PHMA209	Linear And Non-Linear Programming	PHMA309	Greek Means And Arithmetic, Geometric Mean	PHMA409	Real Analysis
PHMA110	Classical Mechanics	PHMA210	Hydrodynamics	PHMA310	Structural Models	PHMA410	Operations Research And C Programming
PHMA111	Functional Analysis	PHMA211	Solid Mechanics	PHMA311	Axiomatic Set Theory	PHMA411	Mathematical Statistics
PHMA112	Combinatorics and Graph Theory	PHMA212	Advanced Discrete Mathematics	PHMA312	Magneto-Hydrodynamics	PHMA412	Mathematical Programming And Design & Analysis of Experiments
-	-	-	-	-	-	PHMA413	Mathematical Techniques in Engineering advanced studies

# GROUP I

PHMA101

## ALGEBRAIC GRAPH THEORY

**Graphs:** Graphs, sub graphs, Automorphisms, Homomorphism, circulant graphs, Johnson graphs, Line graphs, planar graphs.

**Transitive graphs:** Vertex transitive graphs, Edge transitive graphs, edge connectivity, vertex connectivity, matching principle, Hamilton paths and cycles, Cayley graphs.

**Matrix Theory:** Adjacency matrix, incidence matrix, symmetric matrices, Eigen vectors, positive semi definite matrices, sub harmonic functions, Perron-Frobenius Theorem, Rank of a symmetric matrix, spectral decomposition.

**Interlacing:** Interlacing, inside and outside the Peterson graph, equitable partitions, Eigen values of Kneser graphs, more interlacing, Bipartite sub graphs.

**Strongly regular graphs:** Parameters, Eigen values, Some characterizations, Latin square graphs, Small strongly regular graphs, local Eigen values.

**Line graphs and Eigen values:** Generalized line graphs, star-closed sets of lines, reflections, indecomposable star-closed sets, Generating sets, classification, root systems.

**Laplacian of a graph:** Laplacian matrix, trees, representations, energy and Eigen values, connectivity, interlacing, conductance and cut sets, how to draw a graph, Generalized Laplacian.

**Reference:** Algebraic Graph theory by Chris Godsil and Gordon Royle, Springer, New York (2001)

**Classification of second order PDE:** canonical forms, adjoint operators, Riemann's method.

**Elliptic differential equations** – Derivation of Laplace and Poisson equation, Boundary value problems, some important mathematical tools, properties of harmonic functions, separation of variables, Dirichlet problem for rectangle and circle, Neumann problem for rectangle and circle, Mixed boundary value problems.

**Parabolic differential equations** – occurrence, boundary condition, elementary solutions, separation of variables, solution in cylindrical and spherical coordinate systems.

**Hyperbolic differential equations** – occurrence, derivation of one dimensional wave equation, solution of one dimensional canonical reduction, initial value problem, D'Alembert's solution, vibrating string – variable separable method, boundary and initial value problem for two dimensional wave equation – method of Eigen function, uniqueness of the wave equation, Duhamel's principle. **Green's function** – Green's function for Laplace equation, Eigen function method, Green's function for the wave equation, Helmholtz theorem, Green's function for diffusion problems.

**Transform methods:** Laplace transform of Bessel functions, solution of diffusion equations and wave equations by Laplace and Fourier Transform method. Fourier transform for Laplace equation.

**Reference:** Introduction to Partial Differential Equations by K. Sankara Rao, 2<sup>nd</sup> edition.

**Linear Second Order Equations:** Initial value problem, Wronskin, separation and comparison theorems, Poincare phase plane, Adjoint equation, Lagrange identity, Green's function, variation of parameters.

**Boundary value problems:** Sturm Liouville system, Eigen values and Eigen functions, simple properties expansion in Eigen functions, parseval's identity.

**Power series solutions:** Solution near ordinary and regular singular point. Convergence of the formal power series, applications to Legendre, Bessel, Hermite, Laguerre and hyper geometric differential equations with their properties.

**Second order partial differential Equations:** Characteristic curves, reduction to canonical forms, derivation of the equations of mathematical physics and their solutions.

**References:**

1. Theory of ordinary Differential equations, E.A. Coddington and N.Levinson.
2. Methods of Mathematical Physics, Vol.I,II, R. Courant and D.Hilbert
3. Differential Equations with applications and Historical Notes, G.F. Simmons.
4. Theory of partial differential equations, I.N. Sneddon.

Classification of mathematical modeling, Characteristics of mathematical modeling, Population dynamics: Malthusian model, Verhault's model, Simple prey predator model: Lotka Volterra model,

Model on Epidemics (SIS, SIR models only) through systems of ordinary differential equation, Microbial growth in Chemostat (Monod's model)

Industrial Applications:

a) Fermentation Technology b) Conversion of Solar energy c) Ecology and Environment Balance.

Hardy Weinberg's law, Multiple allele finger print systems

Genetic inbreeding models,

Age- structured population Models

a) Discrete- Time Discrete-Age-Scale Population Models.

b) Continuous-Time Discrete-Age-Scale Population Models

Traffic flow model (for traffic density and velocity only), Blood flow in cardiovascular system, The diffusion equation, Control of water pollution , Control of noise pollution.

**References:**

1. Mathematical modeling - J.N. Kapur
2. An Introduction of Bio-statistics - P.S.S. Sundararao
3. Mathematical models in biology-an introduction- Elizabeth S.Allman, John A.Rinodes

**Qualitative Properties and Theoretical Aspects:** Review of Linear Algebra: Vector spaces, concept of linear independence, bases, inner product spaces, linear transformations and matrices, Eigen values and Eigen vectors. Picard's existence and uniqueness theorem: The form of a differential equation, Picard's iteration technique, estimation of the Picard's iterates. Oscillations and the Sturm separation theorem, The Sturm comparison theorem, The Green's function.

**Power series solutions and special functions:** Introduction and review of power series, Series solutions of first order differential equations, second order linear equations; Ordinary points, regular singular points. Gauss's hyper geometric equation, Steady state temperature in a ball.

**Boundary value problems:** Introduction, Eigen values, Eigen functions and the vibrating string. Deviation and solution of wave equation. The heat equation. The Dirichlet problem for a disc – the Poisson integral, Sturm – Liouville problems.

**System of first order equations:** Linear systems, Homogeneous linear systems with constant coefficients. Non-linear systems, Volterra's Prey equations. Solution of system with matrices and exponentials.

**The non-linear theory:** Some motivating examples. Specializing down. Types of critical points – stability, critical points and stability for linear systems. Stability of Liapunov's direct method. Simple critical points of non-linear systems. Non-linear mechanics; conservative systems. Periodic solutions – The Poincare-Bendixson theorem.

**Reference:** Differential Equations by George F Simmons and Steven G Krantz, Tata McGraw Hill Publication.

**High Speed Computation** Introduction, computer Arithmetic, errors in numerical Techniques, machine computation and computer software.

**Transcendental and Polynomial Equations** : Introduction, Rate of Convergence, General Iteration methods, Birge – Vieta, Baristow's, Muller, complex roots, system of non – linear equations by Newton's method.

**System of Linear Algebraic Equations and Eigen value Problems** : Introduction, Direct Methods – Consistency, Rank of a matrix Gaussian elimination, LU decomposition (Crout's algorithm), Cholesky algorithm, Tridiagonal system of equations and Banded system of equations.

**Eigen values and Eigen vectors:** Eigen values and Eigenvectors, bounds on Eigen values, Jacobi's Method for Symmetric Matrices, Given Method for Symmetric Matrices, Householder's Method for Symmetric Matrices, largest Eigen value by power method.

**Interpolation and Approximation:** Introduction, Lagrange and Newton Interpolations, Interpolating polynomials - piecewise polynomial interpolation, Spline's interpolation formula, Hermite Interpolation, Bivariate Interpolation.

**Numerical integration:** Newton's cotes formula, Simpson's rules, Weddle's rule, Gaussian Quadrature, double and triple integration

**Ordinary Differential Equations: Boundary Value Problems:** Initial Value Problem and boundary value problem, Runge – Kutta's Method, predictor – corrector formulae, shooting method for BVP.

**Prescribed Text Book:**

Numerical Methods (Fifth Edition) - M.K. Jain, S.R.K. Iyengar, R.K. Jain, New Age international Publishers

**References:**

- 1) Higher Engineering Mathematics by B. S. Grewal, Khanna publications, 36<sup>th</sup> edition.
- 2) Numerical methods for Engineers by S. C. Chapra and Raymond P. Canale Tata McGraw – Hill Publications Fourth edition.
- 3) K. Shankar Rao, "Numerical Methods for Scientists & Engineers" 2<sup>nd</sup> Edition (2004)



**Fermat and Mercenn's numbers:** Farey series, Farey dissection of the continuum Irrational numbers-Irrationality of  $m$ ,  $N$ ,  $e$  and Approximation Irrational numbers. Hur Witz's Theorem. Representation of a number by two or four squares. A number  $n$  is the sum of two squares if and only if all prime factors of the form  $4m+3$  have even [exponents](#). [tn](#) the standard term of  $n$ . every positive integral is the sum of four squares. Definition  $g(K)$  and  $G(K)$ . Proof of  $g(4) < 50$ , perfect numbers-An even perfect number is of the form  $2^n(2^{n+1}-1)$  where  $2^{n+1}-1$  is prime.

**Continued fractions:** Finite continued fractions, convergent to a continued fraction, continued fractions with positive quotients. Simple continued fractions, The representation of an irreducible rational fraction by a simple continued fraction. The continued fraction algorithm and Euclid's algorithm. The difference between the fraction and its convergence, Infinite simple continued fractions, the representation of an irrational number by an infinite continued fraction, Equivalent numbers periodic continued fractions, some special quadratic surds, and the series of Fibonacci and Lucas.

### References:

1. An introduction to Theory of Numbers, G.H.Hardy and E.M.Wright
2. Elementary Number Theory, Niven and Zuckerman
3. Bruce Burndt-Ramanujan's Note Books Volume-1 to 5.
4. Encyclopedia of Mathematic and its applications, G.E.Andrews-

Metric on a set, pseudo-metrics and metrics Distance between two sets. Equivalent metrics. Limit points and closure: closed sets, Derived set of a set. Adherent points and closure of a set, Densesubsets, Interior of a set and its properties, Subspaces, Product spaces, Structure of Open balls in a product space. Closures and interiors in a product space, Finite product of metric spaces. Convergent sequences. Cauchy sequences. Characterization of adherent points and limit points in terms of convergent sequences. Convergence in products. Convergence in Euclidean spaces. Cluster points of a sequence. Subsequences. Cluster points and convergent subsequences. Algebra of convergent real sequences. Spaces of sequences.

Continuity at a point. Continuity over a space. Continuity of composite, graph and projection maps. Algebra of real valued continuous functions in a metric space. Homeomorphisms. Isometries. Relation between isometries and Homeomorphism. Uniform continuity.

Complete metric spaces. Completeness and Continuous mappings. Completeness and subspaces. Cantor's Intersection Theorem. Contraction Mapping Principle. Connectedness: Connected metric spaces. Connected sets. Characterization of connected subsets of the real line. Properties of Connectedness

Compact spaces and Compact subsets. Compact subsets of the real line. Sequential compactness and its characterization. Countable compactness, Bolzano-Weierstrass property. Sequential characterization of BWP. Equivalence of BWP and sequential compactness. Covering characterization of the BWP. Bolzano-Weierstrass Property and Total boundedness. Bolzano-Weierstrass Property and compactness. Lebesgue covering lemma. Compactness and completeness, Compactness and uniform continuity. Boundedness of continuous real-valued functions on compact metric spaces.

**RECOMMENDED BOOKS:**

1. G. F Simmons: Introduction to Topology and Modern Analysis, McGraw Hill, India
2. E.T Copson: Metric Spaces, Cambridge tracts
3. Dieudonne: Foundation of Modern Analysis, Academic Press, NY
4. Kasriel: Metric Spaces, Wiley, NY

Bessel Functions: Bessel equation and its solution in series, Bessel function, Generating function, Recurrence relations, Integral representation of  $J_n(x)$ , Addition formulae for  $J_n(x)$ , Orthogonality of Bessel functions, Expression of a function in a series of Bessel Functions, Behaviour of  $J_n(x)$  for large values of  $x$ .

Legendre Functions: Legendre equation, Legendre polynomials and Legendre functions, Rodrigue's formula, Generating functions, Recurrence relations, Orthogonality of Legendre Polynomials, Definite integral representation of  $P_n(x)$ , Expansion of a function in a series of Legendre polynomials, behavior of  $P_n(x)$  for large values of  $x$ .

Dirac Delta Functions: Definition of delta function and its properties, Derivative of delta function, Heaviside unit step function and its relation with delta function.

Hyper geometric Function: Hyper geometric differential equation and its solution in series, Different forms of Hyper geometric functions and its simple properties.

Integral Transforms, Laplace Transform, Existence theorem and properties, Laplace transform of functions, Laplace transform of derivatives and integrals, Inverse Laplace transform, Convolution Theorem, Inversion theorem, Solution of differential equations. Fourier Transform : Fourier Integral formula, Fourier transforms and its properties, Fourier transform of simple functions, Relation of Fourier and Laplace Transform, Solution of differential equations. Finite Fourier Transform. Hankel and Mellin Transform : Definition and Examples.

### **References :**

1. Sneddon, I.N. : Special Function of Mathematics, Physics and Chemistry.
2. Sen, B. : Treaties on Special Functions for Scientist and Engineers.
3. Lokenath Debnath : Integral Transform.

Mechanics of a particle, Mechanics of a system of particles, constraints, Generalised coordinates, D'Alembert's principle, Lagrange's equations of motion, the forms of Lagrange's equation for velocity dependent potential, and dissipative forces, applications of Lagrangian formulation, cyclic co-ordinates and generalised momentum, conservation theorems.

Functionals, basic lemma in calculus of variations, Euler- Lagrange's equations, the case of several dependent variables, the minimum surface of revolutions, the problem of Brachistochrone, Isoperimetric problems, Problem of the maximum enclosed area, shape of a hanging rope. Hamilton's principle, Lagrange's equations from Hamilton's principle, (holonomic system) Hamilton's equations of motion from a variational principle. The principle of least action cyclic coordinates and Routh's procedure, conservation theorems and physical significance of Hamiltonian

The two body central force problem : Reduction to the equivalent one body problem, the equations of motion and first integrals, classification of orbits, the virial theorem, the differential equation for the orbit, the Kepler's problem. The kinematics of rigid body motion: The independent co-ordinates of a rigid body, orthogonal transformations, properties of transformation matrix, infinitesimal rotations, the Eulerian angles, the Cayley-Klein parameters, Euler's theorem on motion of rigid body. Angular momentum and kinetic energy of motion of a rigid body about a point, The inertia tensor and moment of inertia, Euler's equations of motion.

### **Recommended Books :**

1. Classical Mechanics by H.Goldstein (1980), Narosa Publishing House, New Delhi
2. Calculus of variations with applications to Physics and Engineering (International series in Pure and Applied Mathematics) by Robert Weinstock (1952) McGraw-Hill book comp. New York.
3. Classical Mechanics by N.C.Rana and P.S. Joag (1991) Tata McGraw Hill, New Delhi.

### **References :**

1. A treatise on the Analytical Dynamics of Particles and rigid bodies. by E.T.Whittaker (1965) Cambridge University Press.
2. Classical Mechanics by E.A.Desolge, Vol. I and II (1982), John-Wiley and sons, New York.
3. Classical Mechanics A Modern Perspective by V.Barger and Martin, Olsson(1995) McGraw Hill, Inc.New York.
4. Classical Mechanics with introduction to Non-linear oscillation and chaos by V.B.Bhatia (1997) Narosa Pub.House.

Bounded linear transformations.  $B(X,Y)$  as a normed linear space. Open mapping and closed graph theorems. Uniform boundedness principle and its consequences. Hahn-Banach theorem and its application. Dual spaces with examples. Separability. Reflexive spaces. Weak convergence. Compact operators. Inner product spaces, Hilbert spaces.

Orthonormal sets, Bessel's inequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Riesz-Fischer theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces. Self-adjoint operators. Positive, projection, normal, and unitary operators.

**References:**

1. Introduction to Topology and Modern Analysis, G. F. Simmons, McGraw-Hill, 1963.
2. Functional Analysis, G. Bachman and L. Narici, Academic Press, 1966.
3. Introduction to Functional Analysis, A. E. Taylor, John Wiley, 1958.
4. Functional Analysis, B. V. Limaye, Wiley Eastern.
5. Linear Operators, Part-I, Interscience, N. Dunford and J. T. Schwartz, 1958.
6. Functional Analysis, R. E. Edwards, Holt Rinehart and Winston, 1965.
7. First Course in Functional Analysis, C. Goffman and G. Pedrick, Prentice- Hall of India, 1987.
8. Functional Analysis and Its Applications, K. K. Jha , Students Friend, 1986

Partially ordered sets, Lattices, Complete lattices, Distributive lattices, Complements, Boolean Algebra, Boolean expressions, Application to switching circuits, Permutations and Combinations, Pigeon-hole principle, Principle of inclusion and exclusion.

Graphs, Vertices of graphs, Walks and connectedness, Degrees, Operations on graphs, Blocks, Cutpoints, bridges and blocks, Block graphs and Cutpoint graphs, Trees, Elementary properties of trees, Center, Connectivity, Connectivity and line connectivity, Menger's theorem, Partitions, Coverings, Coverings and independence number.

**References:**

1. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill, 1986.
2. Kenneth H. Rosen – Discrete Mathematics and its Applications, McGraw-Hill, 2002.
3. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.
4. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
5. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.
6. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2<sup>nd</sup> Ed.,
7. Clark and D. A. Holton – A First Look at Graph Theory, Allied publishers.
8. D. B. West – Introduction to Graph Theory, Pearson Education Inc., 2001, 2<sup>nd</sup> Ed.,
9. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier, 1976.

## GROUP II

### PHMA201

### FLUID DYNAMICS

**Properties of Fluids:** Introduction, properties of fluids, viscosity, thermodynamic properties, Surface tension and Capillarity, Vapor pressure and Cavitation.

**Fluid Statics:** Fluid pressure at a point, Pascal's law, pressure variation in a static fluid, Absolute, gauge, atmospheric and vacuum pressures, simple manometers, differential manometers, total pressure and center of pressure, vertical plane surface submerged in liquid, horizontal plane surface submerged in liquid, inclined plane surface submerged in liquid, curved surface submerged in liquid.

Buoyancy, center of buoyancy, metacenter and metacentric height, conditions of equilibrium of floating and submerged bodies.

**Fluid Kinematics:** Types of fluid flow, Introduction, continuity equation, continuity equation in three dimensions (Cartesian co-ordinate system only), velocity and acceleration, velocity potential function and stream function.

**Dimensional Analysis:** Introduction, derived quantities, dimensions of physical quantities, dimensional homogeneity, Buckingham's  $\pi$  theorem, Raleigh's method, dimensionless numbers, similitude, types of similitudes.

**Fluid Dynamics:** Introduction, equations of motion, Euler's equation of motion, Bernoulli's equation from Euler's equation, Bernoulli's equation for real fluids.

**Fluid flow measurements:** Introduction, venturimeter, orifice meter, Pitot tube. Flow through pipes: Frictional loss in pipe flow, Darcy- Equation for loss of head due to friction in pipes, Chezy's equation for loss of head due to friction in pipes, hydraulic gradient and total energy line.

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**Laminar flow and viscous effects:** Reynold's number, critical Reynold's number, Laminar flow through circular pipe-Hagen poiseuille's equation, Laminar flow between parallel and stationery plates.

#### Text Books:

1. Fluid dynamics by Oijush K. Kundu, IRAM COCHEN, EL SEVIER 3<sup>rd</sup> Ed. 2005.
2. Fluid Mechanics by Dr. Bansal. R. K, Lakshmi Publications, 2004.
3. Fluid Mechanics and hydraulics, Dr. Jagadishlal: Metropolitan Book Co-Ltd, 1997.

**Approximations and Round off Errors:** Significant figures, accuracy and precision, error definitions, round off errors and truncation errors. Mathematical modeling and Engineering problem solving: Simple mathematical model, Conservation Laws of Engineering.

**Roots of Equations:** Bracketing methods-Graphical method,Bisection method,False position method, Newton- Raphson method, Secant Method. Multiple roots, Simple fixed point iteration.

**Roots of Polynomial**-Polynomials in Engineering and Science, Muller's method, Bairstow's Method Graeffe's Roots Squaring Method.

**Numerical Differentiation and Numerical Integration:** Newton –Cotes and Guass Quadrature Integration formulae, ntegration of Equations, Romberg integration, Numerical Differentiation Applied to Engineering problems, High Accuracy differentiation formulae.

**System of Linear Algebraic Equations And EigenValue Problems:** Introduction, Direct methods, Cramer's Rule, Gauss Elimination Method, Gauss-Jordan Elimination Method, Triangularization method, Cholesky Method, Partition method, error Analysis for direct methods, teration Methods.

**Eigen values and Eigen Vectors:** Bounds on Eigen Values, Jacobi method for symmetric matrices, Givens method for symmetric matrices, Householder's method for symmetric matrices, Rutishauser method for arbitrary matrices, Power method, Inverse power method.

**Linear Transformation:** Introduction to Linear Transformation, The matrix of Linear Transformation, Linear Models in Science and Engineering.

**Orthogonality and Least Squares:** Inner product, length and orthogonality, orthogonal sets, Orthogonal projections, The Gram-schmidt process, Least Square problems, Inner product spaces.

**References:**

1. S S Shastri ,Numrical Analysis for Engineers, Tata McGraw Hill Edition
2. M K Jain, S R K Iyengar, R K Jain Numerical Methods for scientific and Engineering computation.



**Introduction to Numerical Methods:** Importance of Numerical Methods in Engineering, Computers, Computer Programming Languages, Data Representation, Programming Structure, Errors, Numerical Methods Considered, Software for Numerical Analysis.

**Solution of Matrix Eigen value Problem:** Introduction, Engineering Applications, Conversion of General Eigen value Problem to Standard Form, Methods of Solving Eigen value Problems, Solution of the Characteristic Polynomial Equations, Jacobi Method, Given's Method, Householder's Method, Eigen values of a Tridiagonal Matrix, Eigenvectors of a Tridiagonal Matrix, Power Method.

**Numerical Differentiation:** Introduction, Engineering Applications, Definition of the Derivative, Basic Finite-Difference Approximations, Using Taylor's Series Expansions, Using Difference Operators, Approximation of Derivatives Using Difference Operators, Using Differentiation of Interpolating Polynomials.

**Numerical Integration:** Introduction, Engineering Applications, Newton-Cotes Formulas, Simpson's Rule, General Newton-Cotes Formulas, Richardson's Extrapolation, Romberg Integration, Gauss Quadrature, Integration with Unequal Segments.

**Ordinary Differential Equations: Initial-Value Problems:** Introduction, Engineering Applications, Simultaneous Differential Equations, Solution Concept, Euler's Method, Improvements and Modifications of Euler's Method, Runge-Kutta Methods, Multi-step Methods, Adams Methods, Predictor-Corrector Methods, Simultaneous Differential Equations, Stiff Equations

**Ordinary Differential Equations and Boundary-Value Problems:** Introduction, Engineering Applications, Shooting Methods, Generalization to  $n$  Equations, Finite-Difference Methods, Solution of Nonlinear Boundary-Value Problems, Solution of Eigen value Problems.

**Partial Differential Equations:** Introduction, Engineering Applications, Initial and Boundary Conditions, Elliptic Partial Differential Equations, Parabolic Partial Differential Equations, Crank-Nicholson Method, Method of Lines, Two-Dimensional Parabolic Problems, Hyperbolic Partial Differential Equations, Method of Characteristics.

**References:**

1. Applied Numerical Methods - Singeresu S. Rao. Pearson Education Inc., 2001.
2. Numerical methods for scientific and engineering computation -M.K. Jain, S.R.K. Iyengar and R.K. Jain New age international publication 5th ed., 2007.

**Some features of Viscous Flows:** Real and ideal fluids, viscosity, Reynolds number, laminar and turbulent flows, asymptotic behavior at large Reynolds numbers.

**Fundamentals of Boundary Layer theory:** Boundary layer concept, laminar boundary layer on a flat plate at zero incidence, turbulent boundary layer on flat plate at zero incidence, fully developed turbulent flow in a pipe, boundary layer on an airfoil, separation of the boundary layer.

**General properties and exact solutions of the boundary layer equations for plane flows:**

Compatibility conditions at the wall, similar solutions of the boundary layer equations, derivation of the ordinary differential equation, boundary layers with outer flow, boundary layers without outer flow, wedge flows, flow in a convergent channel, mixing layer, moving plane, free jet, wall jet, series expansion of the solutions; Blasius series, Gortler series.

**Asymptotic behavior of solutions downstream:** Wake behind bodies, boundary layer at a moving wall, integral relations of the boundary layer; momentum integral equation, energy integral equation, moment of momentum integral equations.

**Approximate methods for solving the boundary layer equations for steady planar flows:**

Integral methods, Stratford's separation criterion. Comparison of the approximate solutions with exact solutions; Retarded Stagnation point flow, Divergent channel (Diffuser), Circular cylinder flow, symmetric flow past a Joukowski airfoil.

**Reference:** Boundary Layer Theory by H. Schlichting and K. Gersten 8<sup>th</sup> revised and enlarged edition, Springer (2000).

Definitions of integral equations and their classification, solution of integral equation, Fredholm integral equations of second kind with separable kernels, solution of Fredholm integral equation with separable kernel, method of successive approximations. Method of successive substitutions, Iterative scheme for Fredholm integral equations of the second kind, resolvent kernel and its results, application of iterative scheme to Volterra integral equations of the second kind.

Conversion of initial value problem to volterra integral equation and conversion of boundary value problem to Fredholm integral equation. Conversion of Fredholm integral equation to boundary value problems and conversion of Volterra integral equation to initial value problem.

Orthonormal system of functions, symmetric kernels, fundamental properties of Eigen values and Eigen functions Green's function, for symmetric kernels, Hilbert Schmidt theory and solutions of Fredholm integral equations with symmetric kernels.

Definition of a boundary value problem for an ordinary differential equation of the second order, Dirac delta function, Green's function, Green's function approach to reduce boundary value problems of a differential equation with homogeneous boundary conditions to integral equations.

**Text Books:**

1. Linear Integral Equation Theory and Techniques by R.P. Kanwal, Academic Press, New York, 1971.
2. Linear Integral Equation (translated from Russian) by S.G. Mikhlin, Hindustan book Agency, 1960.

**Reference:**

1. Boundary value problems of Mathematical Physics by I. Stakgold, Vol.I, II, Mac Millan, 1969.

Topological Spaces, bases for a topology, the order topology, the product topology on  $X \times Y$ , the subspace topology, closed sets and limit points continuous functions, the product topology, the metric topology, the quotient topology.

Connected spaces, connected subspaces of the real line, components and local connectedness, compact spaces, compact spaces of the real line, limit point compactness, local compactness, nets. The countability axioms, the separation axioms, normal spaces, the Urysohn Lemmas, the Urysohn Metrization Theorem, the Tietze Extension Theorem, the Tychonoff Theorem.

**Reference:** 'Topology', by James R. Munkres, second edition 2002.

Vector Spaces: Definition and Examples, Subspaces, Bases and Dimensions, Linear Transformations, Quotient Spaces, Direct Sum, The matrix of Linear Transformation, Duality, Eigen values and Eigenvectors, The minimal Polynomial, Diagonalisability, Triangularisable

Canonical and Bilinear Forms: Jordan Forms, The Rational Forms, Bilinear Forms : Definition and Examples, The matrix of a Bilinear Form, Orthogonality, Classification of Bilinear Forms.

Inner Product Spaces : Inner Product Spaces, Orthogonality, The Adjoint of Linear Transformation, Unitary operators, Self Adjoints and Normal Operators, Polar and Singular Value, Decomposition.

**References:**

1. Algebra by S. Mclane and G. Birkhoff
2. Linear algebra by S. Lang, Springer
3. Linear Algebra by Bisht and Sahai
4. Linear Algebra by Hoffman and Kunze, P.

Tensors: Notations and Summation Convention, Transformation Law for vectors, Cartesian tensors, Algebra of Cartesian tensors, Differentiation of Cartesian tensors, The metric tensor, Transformation of curvilinear co-ordinates, General tensors, Contravariant, Covariant derivative of a vector, Physical components, Christoffel symbol

Curve with Torsion: Tangent, Principal normal, Curvature, Binomial, Torsion, Serret-Frenet formulae, Locus of Center of curvature, Circle of curvature, torsion of a curve, Involutives, Evolutes and Bertrand curves.

Envelopes and Developable Surfaces: Surfaces, Tangent plane, normal, Envelop, Edge of regression, Developable surfaces, Curvilinear co-ordinates on a surface: Fundamental Magnitude. Curve on a Surface : Principal directions and curvature, First and second curvature, Euler's theorem, Normal curvature, Mean curvature, Umbilic points, Conjugate directions and asymptotic lines, Principal directions and principal curvature, Line of curvature and evolute of a surface.

Quadric Surfaces: Central quadrics, Fundamental magnitudes, The first and second fundamental forms, Formulas of Gauss and Codazzi, The fundamental theorem of surface theory, The Gauss-Bonnet Theorem.

**References :**

1. Shanti Narayan : Cartesian Tensors, S. Chand and Company, New Delhi.
2. E.C. Young : Vectors and Tensor Analysis, Marcel Dekker, 1994.
3. A.W. Joshi : Tensors and Riemannian Geometry.
4. C.E. Weatherburn : Differential Geometry.
5. A. Goetz : Introduction to Differential Geometry : Addison Wesley Publishing Company, 1970.

Linear Programming and examples, Convex Sets, Hyperplane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex-method, Charnes-M method, Two phase method, Determination of Optimal solutions, Unrestricted variables, Duality theory, Dual Linear Programming Problems, Fundamental properties of dual problems, Complementary Slackness, Unbounded Solution in Primal. Dual Simplex Algorithm, Sensitivity Analysis.

Parametric programming, Revised Simplex method, Transportation Problems, U-V method, assignment problems, Integer Programming, Gomory's algorithm, Branch & Bound Technique. Game Theory - Two-person, Zero-sum Games with mixed strategies, graphical solution, solution by Linear Programming.

Nonlinear Programming : Convex functions, Unconstrained problems, Kuhn-Tucker conditions for Constrained Optimization, Quadratic Programming methods due to Beale, Wolfe. Duality in Nonlinear Programming, Weak Duality Theorem, Wolfe's Duality Theorem, Hanson-Huard strict converse duality theorem (without proof), Dorn's duality theorem, strict converse duality theorem, Dorn's Converse duality theorem (without proof), Unbounded dual theorem, theorem on no primal minimum.

### **References :**

1. Hadley, G. : Linear Programming, Narosa Publishing House, 6th edition, 1995.
2. Kambo, N.S. : Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd., New Delhi, Madras.
3. Swarup, Kanti, : Operations Research, Sultan Chand & Sons, New Delhi, 9<sup>th</sup> Gupta, P.K. & Man Mohan Edition, 2001.
4. Murty, Katta G. : Linear & Combinatorial Programming, John Wiley & Sons, Inc., 1976 (New York, London, Tronto).
5. Mangasarian, O.L. : Nonlinear Programming, Tata McGraw Hill Company Ltd. (Bombay, New Delhi), 1st Edition, 1969.
6. Bazaraa Mokhtar, S. : Nonlinear Programming, Theory of Algorithms, Wiley, New & Shetty, C.M. York, 1979.

Equation of continuity, Boundary surfaces, streamlines, Irrotational and rotational motions, Vortex lines, Euler's Equation of motion, Bernoulli's theorem, Impulsive actions. Motion in two-dimensions, Conjugate functions, Source, sink, doublets and their images, conformal mapping, Two-dimensional irrotational motion produced by the motion of circular cylinder in an infinite mass of liquid, Theorem of Blasius, Motion of asphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere.

Stress components in real fluid, Equilibrium equation in stress components, Transformation of stress components, Principal stress, Nature of strains, Transformation of rates of strain, Relationship between stress and rate of strain, Navier-Stokes equation of motion. **References:**

1. 1. W. H. Besant and A. S. Ramsey, A Treatise on Hydrodynamics, CBS Publishers and Distributors, Delhi, 1988.
2. 2. S. W. Yuan, Foundations of Fluid Dynamics, Prentice-Hall of India, 1988.



Analysis of Strain–Affine transformation, Infinitesimal Affine deformations, Geometrical interpretation of the components of Strain. Strain Quadric of Cauchy, Principal Strains. Invariants. General Infinitesimal Deformation. Equation of compatibility, Finite deformation.

Analysis of Stress–Stress Tensor, Equations of Equilibrium. Transformation of coordinates. Stress Quadric of Cauchy. Principal stress and Invariants. Maximum normal and shear stresses, Mohr's circle Diagram.

Equations of Elasticity – Generalized Hooke's law. Stress – Strain relations for a medium having one plane elastic symmetry, three orthogonal planes symmetry and for homogeneous isotropic media. Elastic-moduli for isotropic media. Equilibrium and Dynamic equations for an isotropic solids. Strain energy function and its connection with Hooke's law. Unique solution of Boundary value problem. Derivation of Navier's equations and Beltrami-Michal compatibility equations. Statement of problem. Extension of beams by longitudinal forces. Beam stretched by its own weight. Bending of beams by terminal couples. Torsion of a circular shaft. Plane stress. Plane strain.

**References:**

1. S. Timoshenko and N. Goodier, Theory of Elasticity, McGraw-Hill, New York 1970.
2. A.E. Love, A Treatise on the Mathematical Theory of Elasticity, Cambridge University Press, London, 1963.
3. Y.C. Fung, Foundations of Solid Mechanics, Prentice-Hall, New Delhi , 1965.
4. I.H. Shames, Introduction to Solid Mechanics, Prentice-Hall, New Delhi , 1975.
5. S. Valliappan, Continuum Mechanics, Oxford and IBH Publishing Company, New Delhi, 1981.
6. Mathematical Theory of Elasticity, I.S. Sokolnikoff, Tata McGraw-Hill Publishing Company Ltd, 1977,

Formal Logic- Statements, Symbolic Representation of statements, duality, Tautologies and contradictions. Quantifiers, Predicates and Validity of arguments. Propositional Logic. Languages and Grammars, Finite State Machines and their transition table diagrams.

Lattices: Lattices as partially ordered sets, their properties, duality, Lattices as algebraic systems, Sub lattices, Direct products, Bounded Lattices, Complete Lattices, Complemented Lattices and Distributive lattices. Cover of an elements, atoms, join and meet irreducible elements.

Boolean Algebras: Boolean Algebras as lattices. Various Boolean Identities. The Switching Algebra example. Sub algebras, Direct products and Homeomorphisms. Boolean forms and their Equivalence. Min-term Boolean forms, Sum of product Canonical forms.

Minimization of Boolean functions, Applications of Boolean Algebra to Switching Theory (using AND, OR & NOT gates). The Karnaugh Map method. Definition of (undirected) graph, Walk, Path, Circuit, Cycles, Degree of a vertex, Connected graphs, Complete and Bipartite graphs, Planar graphs, Euler's formula for connected Planar graphs, Kuratowski's Theorem (Statement only) and its uses. Colouring of graphs, Five colour theorem and statement of Four colour theorem. Trees , Cut-sets, Spanning Trees, Fundamentals Cut-sets and minimum Spanning Trees, Prim's and Kruskal's algorithms, Connectivity, Matrix Representation of graphs, Directed Graphs, Indegree and outdegree of a vertex.

**References:**

1. **J. P. Trembley & R. Manohar**, Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
2. **J. L. Gersting**, Mathematical Structure for Computer Science (3rd ed.), Computer Science Press, N.Y.
3. **Seymour Lipschutz**, Finite Mathematics, McGraw-Hill Book Co. New –York.
4. **J. E. Hopcroft and J.D. Ullman**, Introduction to Automata Theory Languages & Computation, Narosa Publishing House, Delhi.
5. **C. L. Liu**, elements of Discrete Mathematics, McGraw-Hill Book Co.
6. **N. Deo**, Graph Theory with Applications to Engineering and Computer Sciences, PHI, New Delhi

## GROUP III

### PHMA301

### COMPUTER FUNDAMENTALS AND PROGRAMMING IN C

An overview of functioning of a computer system, Components of a computer system, I/O and auxiliary storage devices ,machine and high level languages, assembler, compiler and interpreters, flow charts and pseudo codes, Basic concepts of operating system.

Introduction to C Essentials – Programs development, Functions. Anatomy of a Function. Variables and Constants Expressions. Assignment Statements, Scalar Data types – Declarations, Different Types of integers. Different kinds of Integer Constants

Floating – point type Initialization, mixing types Explicit conversions – casts Enumeration Types. the void data type, Type definitions. Operators and expression in C-Precedence and associativity, Control flow statements Conditional branching, the switch statement, looping, nested loops, the break and continue statement, the go to statement, infinite loops.

Arrays and multidimensional arrays. Storage classes-fixed vs. automatic duration scope, global variable the register specifier, Functions –user defined and library function,

Introduction to pointers, structures and unions. Introduction to C++: Declaration & Definition of Variables, Data Types, Operators, OOPS Fundamentals: OOPS Versus procedural programming, OOPS terminology, Data abstraction, Data hiding, Encapsulation, Class, Object, Inheritance, Polymorphism.

#### **References:**

1. Computer fundamental by Rajaraman
2. Operating systems concepts by Peterson
3. Programming in ANSI C by E. Balaguruswamy, Tata-McGraw Hill, New Delhi.
4. Programming in C++ by E. Balaguruswamy, Tata-McGraw Hill, New Delhi.
5. Schaum's outline series.
6. Let us C by Y. Kanetkar.
7. Brain W Kernigham & Dennis M Ritchie the C Programmed language, 2nd edition (ANSI features), Prentice Hall 1989.

**Algebra of Complex numbers:** The geometric representation of complex numbers, Elementary theory of power series – sequences, series, convergence, uniform convergence, the Abel's limit theorem, elementary functions.

**The Topology of the complex plane:** Linear transformations, conformal mappings and bilinear transformations.

**Complex Integration:** Cauchy's theorem, Cauchy's integral formula, local properties of analytic functions.

**Taylor's series and Laurent series:** The calculus of residues, harmonic functions, definition and basic properties, the mean value theorem, Poisson's formula, Schwarz's theorem, the reflection principles. Mittag Leffer's theorem, Canonical products, Weirstrass's theorem, The Gamma functions, Jensen's formula.

**References:**

1. Complex Analysis by Ahlfors, McGraw Hill Publications,
2. Functions of one complex variable by Conway, Narosa publications, New Delhi.

- > Basic concepts of fluid dynamics
- > The rheology of human blood
- > The properties of blood vessels
- > Cardiovascular system and blood flows.
- > Steady non Newtonian fluid flows in circular tubes
- > Newtonian Pulsatile flows in rigid and elastic tubes
- > Blood flow through artery with mild stenosis
- > Peristaltic flows in tubes and channels
- > Models for gas exchange and air flow in lungs
- > Two-dimensional flow in retinal tube
- > Lubrication of human joints
- > Diffusion equation
- > Diffusion in artificial kidney (hemodialyser)
- > Mutation and reversion in Bacterial growth
- > Sampling theory and Test of significance: F-Test and ANOVA table

**References:**

1. Bio Fluid mechanics- Jagan N. Mazumdar
2. Introduction to Mathematical Biology- S.I. Rubinow
3. Text book of Medical Physiology- Guyton and Hall
4. Mathematical Models in Biology and Medicine- J.N. Kapur
5. Transport phenomena in the Cardiovascular system- Stanley Middleman

RANDOM EVENTS AND RANDOM VARIABLES: Random events - Probability axioms - Combinatorial formulae - conditional probability - Bayes Theorem - Independent events - Random Variables - Distribution Function - Joint Distribution - Marginal Distribution - Conditional Distribution - Independent random variables - Functions of random variables.

PARAMETERS OF THE DISTRIBUTION: Expectation- Moments - The Chebyshev Inequality - Absolute moments - Order parameters - Moments of random vectors - Regression of the first and second types.

CHARACTERISTIC FUNCTIONS: Properties of characteristic functions - Characteristic functions and moments - semi-invariants - characteristic function of the sum of the independent random variables - Determination of distribution function by the Characteristic function - Characteristic function of multidimensional random vectors - Probability generating functions.

SOME PROBABILITY DISTRIBUTIONS: One point , two point , Binomial - Polya - Hypergeometric - Poisson (discrete) distributions - Uniform - normal gamma - Beta - Cauchy and Laplace (continuous) distributions.

LIMIT THEOREMS: Stochastic convergence - Bernoulli law of large numbers - Convergence of sequence of distribution functions - Levy-Cramer Theorems - de Moivre-Laplace Theorem - Poisson, Chebyshev, Khintchine Weak law of large numbers - Lindberg Theorem - Lyapunov Theroem - Borel-Cantelli Lemma - Kolmogorov Inequality and Kolmogorov Strong Law of large numbers.

**References:**

1. R.B. Ash, Real Analysis and Probability, Academic Press, New York, 1972
2. K.L.Chung, A course in Probability, Academic Press, New York, 1974.
3. R.Durrett, Probability:Theory and Examples, (2nd Edition) Duxbury Press, New York, 1996.
4. V.K.Rohatgi An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern Ltd., New Delhi, 1988(3rd Print).
5. S.I.Resnick, A Probability Path, Birhauser, Berlin,1999.
6. B.R.Bhat , Modern Probability Theory (3rd Edition), New Age International (P)Ltd, New Delhi, 1999
7. M. Fisz, Probability Theory and Mathematical Statistics, John Wiley and Sons, New York, 1963.

**Basic Differential equations** - The Navier-Stokes Equations, The Generalized Reynolds equation, Flow and Shear Equations, Derivation of Energy Equation, Equation of state.

Incompressible Lubrication; One dimensional Bearings - the real Bearing, One-dimensional Journal Bearings, Infinitely Long Bearing, Infinitely Short Bearing, Partial Bearings, fitted Bearings, Floating-ring Bearings, Pours Bearings .

**Incompressible Lubrication; Finite Bearings** – Introduction, Finite Journal Bearings, Analytical Methods, Journal Bearing with Axial Feeding, Journal Bearing with Circumferential Feeding, Numerical Methods, Digital Computers, Electrical Analogues Journal Bearing Solutions, Full Journal Bearings, Centrally Loaded Partial Bearings, Axial Groove Bearings, Non Circulr Bearings, Finite Thrust Bearings with Analytical solutions.

**Squeeze Film and Dynamic Loading:** Dynamically Loaded Bearings , The Reynolds Equation for Dynamic Loading Cyclic Squeeze Films in Journal Bearings, Constant Loads, Alternating Loads, Non-cyclic Squeeze Films, Journal Bearings Spherical Bearings, Conical Seats, Sliders and Rectangular Plates, Elliptical and Circular Plates, Dynamic Loading, Squeeze Films .

**Non-Newtonian Fluids:** General Remarks Bingham Plastics (Greases) as Lubricants, Rheodynamic Bearings, Squeeze Films, Rheostatic Bearings, and Visco-elastic Lubricants.

**Reference:** Theory of Hydrodynamic Lubrication by Oscar Pinkus, McGraw-Hill publications.

**Basic set theory** – detailed discussion on set operators. Cartesian product, properties of product sets, inclusion-exclusion principle. Relations and functions – introduction, properties, equivalence relations and partitions.

**Order relations** - ordered sets, representation of a ordered set, lattices, distances in a set. Graphs and paths. Introduction, representation of a graph, The matrices associated with a graph, chains and cycles, Hamilton paths and cycles, Arborescence.

**Lengths and derivatives in a graph** – introduction, finding the deviations. The making method to compute the deviations from one point to all other points, deviation matrix, matrix theory, powers of a square matrix, the matrix method for the computation of deviations, finding the tracks between two points.

**Connectivity of a graph and Kindred Notions** – Introduction, types of connectivity, finding the maximal strongly connected components of a graph by marking (Roy's algorithm), degree of connectivity, cliques, stability and kernel of a graph. Articulation point and set, power of a point.

**The set of  $n$  – point graphs** – Introduction, Isomorphism, lattices of  $n$  – points graphs, Sigma equivalence lattices of  $n$ - points symmetric graphs, Distance between graphs, Various types of graphs, Simple graphs, valued graph, Multi graph, Algebraic graph, Marked graph.

Balancing Process- Definition of the Balance of the balance of a graph, Other types of Balance, Characteristics of balanced graph, Degree of Unbalance of a graph.

**Reference:** Applications of Graph Theory to Group structures by C. Flament, Prentice – Hall publications, Englewood Cliffs, NJ



**Pipe and Channel Flows:** Introduction, Laminar flow theory, Channel and pipe, Reynolds number, entry length, transition to turbulent flow, relationship between flow rate and pressure gradient.

**Flow past a circular cylinder:** Introduction, Reynolds number, flow patterns, drag.

**Equations of Motion:** Introduction. Fluids particles and continuum mechanics. Eulerian and Lagrange coordinates, continuity equation, substantive derivative, the Navier – Stokes equation, boundary conditions, conditions for incompressibility.

**Some solution of the viscous flow equations:** Introduction, Poiseuille's flow, rotating coquette flow, Stokes flow past a sphere, low Reynolds number flow past the cylinder.

**Inviscid flow:** Introduction, Kelvin's circulation theorem, irrotational motion, Bernoulli's equations. Drag in inviscid flow; D'Alembert's paradox, Applications of Bernoulli equations.

**Thermal flows:** Introduction, equations of convection, classification of convective flows, forced convection, flow with concentration variations (mass transfer).

**Free convection:** Introduction, the governing non-dimensional parameters, the adiabatic temperature gradient, free convection as a heat engine. Convection from a heated vertical surface, thermal plumes, convection in fluid layers.

**Experimental methods:** General aspects of experimental fluid dynamics, velocity measurement, pressure and temperature measurement, flow visualization.

**Practical situation:** Introduction, cloud patterns, waves in atmospheric circulation. Continental drift and convection in the earth mantle. Solar granulation. Effluent dispersal, wind effects on structures. Boundary layer control; vortex generators. Fluidics, and Undulatory swimming, convection from a human body, the flight of a boomerang.

**Magneto hydrostatics:** Pinch confinement of plasma, Equilibrium of sun spot.

**Magneto Hydrodynamic waves:** Alfvén waves. Reflection and transmission of Alfvén waves at a discontinuity in density. Magneto Hydrodynamic shock waves – shock wave in non-conducting gas with finite viscosity and thermal conductivity. MHD effects in shock formation. Laminar flows of viscous conducting liquids between parallel plates in a transverse magnetic field.

### References:

1. Bio Fluid Mechanics by Jagan N Mazumdar
2. Viscous Fluid Dynamics by J. L. Bansal
3. Mathematical Models in Biology and Medicine by J. N. Kapur.

Formal Logic-Statements, Symbolic Representation and Tautologies, Quantifiers, Predicates and Validity, Propositional Logic, Lattices: Partially ordered sets and Lattices, Hasse Diagrams, lattices as algebraic systems `sub-lattices, direct product and Homomorphisms, Complete lattices, Modular lattices, distributed lattices, the complemented lattices, convex sub lattices, Congruence relations in lattices.

Conversion of Boolean Algebra into Boolean rings and vice versa, Boolean Algebras as Lattices, Various Boolean identities, The Switching Algebra, Sub algebras, Direct Products and Homomorphism, Join irreducible elements, Atoms and Minterms, Boolean Forms, Applications of Boolean algebra to Switching Theory

Graphs, complete graphs, regular graphs, bipartite graphs, Vertex degree, subgraphs, paths and cycles, the matrix representation of graphs, fusion, trees and connectivity, bridges, spanning trees, connector problems, shortest path problems, cut vertices and connectivity.

**References:**

1. Elements of Discrete Mathematics by C. L. Liu, McGraw-Hill Book Co.
2. Discrete mathematical structures by Kolman, Busby and Ross, 4th edition. PHI, 2002.
3. Mathematical Structures for Computer Science by J. L. Gersting, (3rd edition), Computer Science Press, New York.
4. Discrete Mathematics with Graph Theory by Goodaire and Parmenter, Pearson edition. 2nd edi.
5. Graph Theory with Applications to Engineering and Computer Sciences by N. Deo, PHI
6. Discrete Mathematics by Harikrishna, Sandeep Kumar, Pragati Prakashan.

Introduction to Greek means, Definition and properties of means, comparison of means, weighted Greek means, invariant and complementary means, partial derivative and means.

Double sequences: Measurement of the circle, Heron's method of extracting square roots, Lagrange and the definition of the AGM, Elliptic integrals, Gaussian double sequences, determination of G compound means, Archimedean Double sequences.

**Reference:** Gheroghe Toader and Silvia Toader, RGMIA Monograph, Australia (2005).

**Digraphs and structures:**

Definition, nets and relations, properties of relations, digraphs, kinds of digraphs, implication digraphs, digraphs and matrices, isomorphism of digraphs, structural models.

**Joining and reaching:**

Semi paths and paths, directional duality, principle of directional duality, the joining of pairs of points, realization of reach ability and joining, unipathic digraphs.

**Diagraphs and Matrices:**

Matrix operations, the reach ability matrix, limited reach ability, uses of reach ability matrix, strong components of D, point bases and fundamental sets, kinds of connectedness, the connectedness matrix, distance matrix, how to find the geodesic sub graph from one node to another node, the number of paths of given length.

**Balancing structures:**

Criteria for balance, local balance measure of structural balance, the line index for balance, limited balance, cycle balance and path balance.

**Networks:**

Definition, the matrix of a network, Markov chains, cost geodesic, flows in networks.

**Reference:** Structural models: An introduction to the theory of directed graphs by Frank Harary, Robert Z Norman, D Cartwright, John Wiley and sons Inc, NY.

Set theory and foundations of mathematics, logics and notations, axiom schema of abstraction and Russell's paradox. Axiom of extensionality and separation, intersection, union and differences of sets, pairing axiom and ordered pairs. Definition by abstraction, sum axiom and families of sets, power set axiom, Cartesian product of sets, axiom of regularities.

Operation on binary relations, ordering relation, equivalence relation and partitions, functions, equipollence, finite sets.

Cardinal numbers, finite cardinals, definitions and general properties of ordinals, finite ordinals and recursive definition, denumerable sets, transfinite induction and definition by transfinite recursion. Elements of ordinal arithmetic, well ordered sets, axioms of choice, Zorn's Lemma, Hausdorff maximal principle, law of trichotomy and their equivalences.

**Reference:** Patric Suppes, Axiomatic Set Theory, East-West Press New Delhi, 1975

**Basic equations** – outline of basic equations of MHD including Faraday's laws and constitutive laws.

Magnetic induction equation – Lorentz force – MHD approximations. Non-dimensional numbers – velocity, temperature and magnetic field boundary conditions.

**Exact equations** – Hartmann flow – isothermal boundary conditions – temperature distribution in Hartmann flow – Hartmann couette flow.

**Applications** – Concepts in Magnetostatics. Classical MHD and Alfven's wave, Alfven's theorem, Frozen – n – phenomenon and equipartition of energy by Alfven's wave.

**Classification of convection** – Free, forced and mixed. Free convective motion of viscous fluids – Analytical solution for the Rayleigh – Benard convection in a Newtonian fluid. Analytical solution for the Benard – Marangoni convection problem in a Newtonian fluid. Galerkin solution of Rayleigh – Benard convection problem for fluid – rigid boundaries. Galerkin solution of Rayleigh – Benard convection problem for rigid – free boundaries. Galerkin solution of Rayleigh – Benard convection problem for free – free boundaries subjected to isothermal and adiabatic conditions.

**References:**

1. Fluid Mechanics by F. Chorlton & Von Norstrang.
2. Hydrodynamic and hydro magnetic stability by S. Chandrasekhar, Dover's university.

Introduction, magnetic fluids, concepts of fluid mechanics, equation of continuity, substantial derivative, generalized Bernoulli equation

magnetic fluids: stability requirements, stability in magnetic field gradient, stability against settling in a gravitational field, stability against magnetic agglomeration, preparation of ferro fluid by chemical precipitation, magnetic precipitation with steric stabilization, cobalt particles in an organic carrier, other magnetic fluids, paramagnetic salt solution, metallic based ferrofluid, ferrofluid modifications, physical properties, equilibrium magnetization.

Electromagnetism and fields: magnetostatic field equation, magnetic field boundary condition, Maxwells equations, integral equations, differential equations, equations of motion for magnetic fluid and thermodynamics of magnetic fluids.

Ferrohydrodynamic instabilities: Normal field instability, interfacial force balance, kinematics , flow field analysis, perturbed magnetic field, boundary conditions on field vectors, porous medium flow, fingering instability, thermocovective stability.

**Reference** : Ferrohydrodynamics by R.E.Rosenweig, Cambridge University Press, London 1985

Functionals, Linear Functionals, Fundamental Lemma of Calculus of Variations Simple Variational Problems, The Variation of Functional, the Extremum of Functional, Necessary Condition for Extreme, Euler's Equation, Euler's Equation of Several Variables, Invariance of Euler's Equation, Motivating Problems of Calculus of Variation, Shortest Distance, Minimum Surface of Revolution, Brachistochrone Problem, Isoperimetric Problem, Geodesic. The Fixed End Point Problem for 'n' Unknown Functions, Variational Problems in Parametric form, Generalization of Euler's Equation to (i) 'n' Dependent Functions (ii) Higher Order Derivatives

Variational Problems with Subsidiary Conditions, Derivation of the Basic Formula, End Points Lying on Two Given Curves or Surfaces.

Integral Transform: Laplace Transform; Transform of Elementary Functions, Transform of Derivatives, Inverse Transform, Convolution Theorem, Applications: Ordinary and Partial Differential Equations.

Fourier Transform, Sine and Cosine Transform, Inverse Fourier Transform, Application to Ordinary and Partial Differential Equations.

**References:**

1. I. M. Gelfand & S. V. Fomin: Calculus of Variations, Prentice-Hall.

Chapter -III (13,14) Chapter -2 (9,10,11,12) .

2. L Debnath : Integral Transforms and their Applications, CRC Press, 1995.

3. R. Churchill, Operational Mathematics, McGraw Hill, New York 1972.

4. I. N. Sneddon: The use of Integral Transform, McGraw Hill, New York 1972.



**Optimization Models in Biology:**

- > Models for blood testing and patient care
- > Dorfman procedure
- > Sterett procedure
- > Sobel- Groll procedure
- > Dynamic programming formulation for patient care
- > Water quality management
- > Noise pollution control models
- > Linear programming Model- Graphical Method, Simplex Method, Duality in Linear Programming

**Applications of Partial Differential Equations:**

- > classification of second order partial differential equations
  - > Canonical forms- Hyperbolic, Parabolic, Elliptic Equations
  - > Laplace Transform: Periodic functions
  - > Error function
  - > Bessel's function
  - > Faltung theorem
  - > Solution of Diffusion Equation
  - > Transform of Unit step function
  - > Solution of Ordinary Differential Equations >
- Solution of Partial Differential Equations

**References:**

1. Introduction to Partial Differential Equations- K. Sankara Rao
2. Mathematical Models in Biology and Medicine- J.N. Kapur
3. Optimization theory and applications- S.S Rao

**Basics of Lubrication** – Introductory, Converging Wedge, Petroff's law (no derivation), a note on the use of non-dimensional quantities, Viscosity – definition, Kinetic viscosity.

**Reynolds' Equation** – Introduction, Assumption, Equilibrium of an element, Lubricant flow, Reynolds' equation for infinitely long bearing, Reynolds' equation in two and three dimensions, simple derivation, Reynolds' equation in three dimensions- Introduction, Continuity of flow of an element, Continuity of flow of a column, full equation, Simplification of the equation, Full Reynolds' equation- Discussion, General methods of the solution, Infinitely long bearing, Infinitely short bearing, Reynolds' equation in polar coordinates.

**Journal Bearings** – Introduction, Oil film thickness, approximate relation, Film shape, accurate expression, Reynolds' equation for journal bearings, Pressure equation, Boundary condition, Sommerfeld number, half Sommerfeld condition, The Reynolds condition, Narrow bearing. Attitude angle and oil flow, Friction in journal bearing, Friction with cavitations, Coefficient of friction.

**Finite Journal Bearing** - Introduction, angular extent of bearings, Oil film thickness, Reynolds' equation for journal bearings, Pressure and angles, Load. Sommerfeld number, Oil flow, circumferential flow, axial flow, Flow from a groove, Flow from a hole.

**Porous Metal Bearing** - Introduction, Physical characteristics of porous metals – Porosity, Oil content, Degree of sintering, Radial strength, Tensile strength, Permeability, Stress-strain characteristics, other physical properties, Porous bearing theory, Bearing performance – Basic mechanism, Temperature, Clearance, P.V. factor, Amount of under design, Quality of the oil, Life, Supplementary lubrication, Fitting, Limitations and advantages of porous metal bearings.

**References:**

1. Theory of Hydrodynamic Lubrication by Oscar Pinkus, McGraw-Hill publication
2. The Principles of Lubrication by A Cameron, Longmans Green & Co Ltd, London

Product Of Vectors, Differentiation Of Vectors (Gradient, Divergence, Curl), irrotational and Solenoidal cases, Integration of vectors (Green's, Stoke's and Gauss Divergence theorem)

Real fluids and ideal fluids, velocity of fluid at a point, streamlines, pathlines, streaklines, velocity potential, vorticity vector, local and particle rate of change, equation of continuity, irrotational and rotational motion, acceleration of fluid, conditions at rigid boundary.

Euler's equation of motion, Bernoulli's equation, Potential theorems, axially symmetric flows, impulsive motion, Kelvin's Theorem of circulation, equation of vorticity. Some three dimensional flows, sources, sinks and doublets, images in rigid planes, images in solid sphere. Stoke's stream function. Two Dimensional Flows: Complex velocity potential, Milne Thomson Circle Theorem, Theorem of Blasius, Vortex rows, Karman Vortex Street.

Viscous Flows: Stress components, Stress and strain tensor, Coefficient of viscosity and Laminar flow, Plane Poiseuille flows and Couette flow. Flow through tubes of uniform cross section in the form of circle, Ellipse, equilateral triangle, annulus, under constant pressure gradient. Diffusion of Vorticity. Energy dissipation due to viscosity, steady flow past a fixed sphere, dimensional analysis, Reynold numbers, Prandtl's boundary layer. Boundary layer equation in two dimensions, Karman integral equation. Elements of wave motion; waves in fluids : Surface gravity waves, group velocity, energy of propagations, path of particles, waves at interface of two liquids.

**References :**

1. Chorlton, F: Text Book of Fluid Dynamics, 1st Edition.
2. Landau, L. D. & Lipschitz, E. N. : Fluid Mechanics, Fluid Dynamics, Pergamon, New York, 1987.
3. Batchler, G. K: An Introduction to Fluid Mechanics, Cambridge University Press.
4. Besant, W. H. and Ramsey, A. S: A Treatise on Hydromechanics.

**Models for Bio fluid flows (to set three questions only)**

- > Navier Stokes Equation for Steady incompressible flow
- > Couette Flow for different cases.
- > Applications of Poiseuille's flow for blood flow
- > Pulsatile flow
- > Mones- Korteweg expression of wave velocity in an inviscid Fluid-Filled Elastic cylindrical tube

**Non Newtonian fluids(to set two questions only)**

- > Visco elastic fluids
- > Laminar flow of Non – Newtonian fluids
- > Power law model
- > Herschel Bulkley Model

**Models for other flows(to set three questions only)**

- > Fluid flows in kidneys- Introduction
- > Diffusion process in Haemodialyser.
- > Peristaltic flows- Peristaltic motion in a cylindrical tube
- > Long wavelength analysis
- > Velocity Distribution
- > Volume flow rate
- > Temperature Distribution at the wall

**References:**

1. Bio Fluid mechanics- Jagan N. Mazumdar
2. Viscous Fluid Dynamics- J.L. Bansal
3. Mathematical Models in Biology and Medicine- J.N. Kapur

**Fundamental concepts**

1. classification of second order partial differential equations
2. Canonical forms- Hyperbolic, Parabolic, Elliptic Equations
3. Adjoint operators
4. Riemann Method

**Laplace Transform Methods**

1. Transforms of Derivatives, Differential equation and simultaneous differential equations.
2. Transform of Dirac Delta function
3. Inverse Transform
4. Integral Equations
5. Solution of Diffusion Equation
6. Miscellaneous examples

**Fourier Transform Methods**

1. Fourier Integral Theorem
2. Sine and Cosine Integral representations
3. Transform of Elementary functions
4. Properties of Fourier Transforms
5. Faltung Theorem
6. Parseval's Relation
7. Transform of Dirac Delta function
8. Solution of Diffusion Equation
9. Solution of Wave Equation
10. Solution of Laplace Equation

**Reference:** 1. Introduction to Partial Differential Equations- K. Sankara Rao 2.

Higher Engineering Mathematics, B S Grewal.

Axiomatic approach to probability, combinatorial problems. Conditional probability and independence. Random variables, expectation and moments. Discrete and continuous distributions. Binomial, Poisson, Normal, Uniform, Beta, Gamma, Chi-square and Bivariate normal distribution. Sampling distribution of mean and variance (normal population).

Chebyshev's inequality, weak law of large numbers, De-Moivre—Laplace and Lindeberg—Levy Central Limit theorems.

Descriptive Statistics : Introduction to statistical methods, frequency distributions, measures of central tendency and dispersion, moments and measures of Skewness and Kurtosis. Fitting of Binomial, Poisson and Normal distribution, Theory of attributes independence and association, bivariate correlation and regression.

General concept of Point estimation, unbiasedness, consistency, efficiency, sufficient statistics, Factorization Theorem, Completeness Rao-Blackwell Theorem, Cramer Rao Inequality. Maximum Likelihood method of estimation and method of moments, Interval estimation, confidence intervals for means, difference of means and variances. Testing of Hypotheses, uniformly most powerful tests, likelihood ratio tests. t, Chi-square and F-distribution. Tests of significance based on t, Chi-square and F. Analysis of variance.

**References :**

1. Hogg, R.V and : Introduction to Mathematical Statistics, MacMillan, 2002.
2. Goon, A.M. : Fundamental of Statistics, Vol. 1, 7th Edition, 1998.
3. Fisz, M. : Probability and the Mathematical Statistics.
4. Feller, W. : Introduction to Probability and its Applications, Vol. 1.

Definition and existence of Riemann-Stieltjes integral, Conditions for R-S integrability. Properties of the R-S, integral, R-S integrability of functions of a function. Series of arbitrary terms. Convergence, divergence and oscillation, Abel's and Dirichlet's tests. Multiplication of series. Rearrangements of terms of a series, Riemann's theorem. Sequences and series of functions, pointwise and uniform convergence, Cauchy's criterion for uniform convergence. Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation. Weierstrass approximation theorem. Power series. Uniqueness theorem for power series, Abel's and Tauber's theorems.

Functions of several variables. Derivative of functions in an open subset of  $\mathbb{R}^n$  into  $\mathbb{R}^m$  as a linear transformation. Chain rule. Partial derivatives. Taylor's theorem. Inverse function theorem. Implicit function theorem. Jacobians. Measures and outer measures. Measure induced by an outer measure, Extension of a measure. Uniqueness of Extension, Completion of a measure. Lebesgue outer measure. Measurable sets. Non-Lebesgue measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Integration of non-negative functions. The general integral. Convergence theorems. Riemann and Lebesgue integrals.

### **References:**

3. 1. Walter Rudin, Principle of Mathematical Analysis (3rd edition) McGraw-Hill Kogakusha, 1976,
4. International Student Edition.
5. 2. K. Knopp, Theory and Application of Infinite Series.
6. 3. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.

Game Theory: Two person zero sum games, Games with mixed strategies, Graphical solution, Solution by linear programming. Basic Concept of Multi Objective and Multi Level Optimization. Integer Programming, Mixed Integer Programming. Linear Fractional Programming. Goal Programming. Sensitivity Analysis and System Reliability.

Geometric Programming: Constrained and Unconstrained Minimization Problems.

Dynamic Programming: Deterministic and Probabilistic dynamic programming.

Stochastic Programming: Stochastic Linear and Stochastic Non linear Programming.

Network Scheduling by PERT/CPM.

**Programming in C:** Basic concepts of programming languages: Programming domains, language evaluation criterion and language categories, Describing Syntax and Semantics, formal methods of describing syntax, recursive descent parsing, attribute grammars, Dynamic semantics (operational semantics, denotational semantics, axiomatic semantics)

Names, Variables, Binding, Type checking, Scope and lifetime data types, array types, record types, union types, set types and pointer types, arithmetic expressions, type conversions, relational and Boolean expressions, assignment statements, mixed mode assignment. Statement level control structures, compound statements, selection statement, iterative statements, unconditional branching.

### **References:**

1. F. S. Hiller and G. J. Lieberman, *Introduction to Operations Research* (6th Edition), McGraw-Hill, International Edition, 1995.
2. G. Hadley, *Nonlinear and Dynamic Programming*, Addison Wesley.
3. H. A. Taha, *Operations Research – An Introduction*, Macmillan.
4. Kanti Swarup, P. K. Gupta and Man Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi.
5. S. S. Rao, *Optimization Theory and Applications*, Wiley Eastern.
6. N. S. Kambo, *Mathematical Programming Techniques*, Affiliated East-West Press Pvt. Ltd., New Delhi.
7. Concepts of Programming Language by Robert W. Sebesta, Addison Wesley, Pearson Education Asia, 1999.
8. How to Program C by Deitel and Deitel, Addison Wesley, Pearson Education Asia.



**Distributions of Random Variables**

The probability set Function, random variables, The probability density Function, the distribution function, Certain probability Models, Mathematical Expectation, Some special Mathematical expectations, Chebyshev's Inequality, conditional probability, Marginal and conditional distributions, the correlation coefficient, Stochastic Independence.

**Some Special Distributions:** The Binomial, trinomial, and Multinomial Distributions, the Poisson Distribution, The Gamma and Chi-square Distributions, the normal distribution, and the bivariate normal distribution. Sampling theory, Transformations of variables of the Discrete type, Transformations of the variables of the continuous type. The t and F distributions. Extensions of the change-of-variable Technique, Distributions of order statistics, the moment generating function Technique, The distribution of  $c$  and  $nS^2/\sigma^2$ , Expectations of Functions of Random variables, Limiting Distributions, Stochastic Convergence, Limiting Moment Generating Functions, The Central limit Theorem, some theorems on limiting Distributions.

**Reference:**

1. Feller, W, Introduction to Probability and its Applications, Wiley Eastern Pvt. Ltd. Vol. 1, (1972).
2. Robert V. Hogg and Allen T. Craig, Introduction to Mathematical Statistics, Forth Edition, Macmillan Publishing Co., Inc., New York, 1989, (Chapter 1 to 5).

Linear Programming, optimal solutions of linear programming problems. Duality in linear programming. The dual Simplex method. Integer Programming: Importance of integer programming problems.

Dynamic programming, Application of dynamic programming, Bellman's principle of optimality, solution of problems with a finite number of stages, Inventory control and solving linear programming problems.

Network Analysis: Shortest path problem. Minimum Spanning Tree Problem. Maximum flow problems.

Network Simplex Method. Project planning and Control with PERT-CPM.

Non linear programming problems: The general Linear Programming problems of constrained maxima and minima. Quadratic Programming: General quadratic programming problem, Kuhn Tucker conditions of quadratic programming problems, Examples based on Wolfe's method and Beale's method

Concepts of Experiments, Principles of Design of Experiments, Analysis of Non-Orthogonal Data, Two Way Data, Complete Block Designs. Factorial Experiments: Symmetrical, Asymmetrical factorial designs. Confounding, Partial-Confounding and Fractional Factorial Designs. Incomplete Block designs and their analysis. BIBD, PBIBD Construction and Analysis.

Response Surface Methodology.

**References:**

1. Kempthorne: The Design and Analysis of Experiments, John Wiley
2. Das & Giri: Design and Analysis of Experiments, Wiley Eastern
3. Day: Incomplete Block Designs, Wiley Eastern
4. Montgomery: Design and Analysis of Experiments, John Wiley
5. S. D. Sharma: Operations Research, Kedar Nath Ram Noth and co.
6. Kanti Swarup, P.K.Gupta and Manmohan : Operations research, S. Chand & Co.
7. Hamady Taha: Operations Research, Mac Millan Co
8. S. D. Sharma: Nonlinear and Dynamic programming, Kedar Nath Ram Nath and Co.
9. G. Hadley: Linear programming, Oxford and IBH Publishing Co.
10. S. I. Gass: Linear Programming, Mc Graw Hill Book Co

## PHMA413: Mathematical Techniques in Engineering advanced studies

### Linear algebra and Fourier transforms

**Linear algebra.** Linear and orthogonal transformations, Eigen values and Eigen vectors, Reduction to diagonal form, Reduction of quadratic and canonical forms. Hermitian matrix, Unitary matrix.

**Fourier transform.** Infinite Fourier transform, Fourier sine and cosine transforms, fast Fourier transform.

### Statistics & Probability.

**Statistics.** Curve fitting by the method of least squares,  $y=a+bx$ ,  $y=a+bx+cx^2$ ,  $y=axy$ ,  $y=abx$ ,  $y=aebx$  Correlations and regression.

**Probability.** Random variables, Binomial, Poisson, Uniform, exponential, normal distribution and Chi square distribution.

### Partial differential equations

Formation of PDE, solution of non-homogenous, method of separation of variables. Solutions of Lagrange's linear PDE of the type  $Pp+Qq=R$

### Optimization and MATLAB

**Optimization.** Optimization problem, Linear programming problem, mathematical formulation, Graphical method and Simplex method, Degeneracy and difficulties in starting.

**An introduction to MATLAB.** Programming in MATLAB. Polynomials, curve fitting and interpolation. Applications in Numerical Analysis. Verification of the results for solving the examples using Numerical methods and using MATLAB.

### Text Books:

1. Higher Engineering Mathematics , B.S.Grewal 39th edition kanna publications,2005.
2. MATLAB An introduction with applications, Amos Gilat.

### References:

1. Advanced Engineering mathematics Erwin Kreyszig
2. David C Lay, Linear algebra and its applications , 3rd edition, Pearson Education, (Asia) Pte, Ltd 2005.
3. Applied numerical methods with MATLAB, 2 e, Steven Chapra, McGraw- Hill company Ltd, New Delhi

